

# Mechanical Design, Simulation and Nonlinear Control of a New Exoskeleton Robot for Use in Upper-Limb Rehabilitation after Stroke

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**Abstract**— Cerebrovascular accident (CVA) or stroke is one of the main causes of disability. It affects millions of people worldwide. One symptom of stroke is disabled arm function. Restoration of arm function is necessary to resuming activities of daily living (ADL). Along with traditional rehabilitation techniques, robot-aided therapy has emerged in recent years. Robot-aided arm therapy is more intensive, of longer duration and more repetitive. By using robots repetitive dull exercises can turn into a more challenging and motivating tasks such as games. Besides, robots can provide a quantitative measure of the rehabilitation progress. This paper introduces a new robot for shoulder rehabilitation. The shoulder rehabilitation system (S.R.S) has three degrees of freedom (DOFs) for three rotational degrees of freedom of the shoulder. It also allows the additional translational DOFs of the shoulder to avoid discomfort to the patient. A new open circular mechanism is proposed for the third joint. The mechanical structure is designed and optimized in Solidworks and it is based on the properties of upper limb of an adult person. The proposed control algorithm is inverse dynamics control which is intended to be used in passive rehabilitation. The proposed control can efficiently track the desired trajectory and reject constant bounded disturbance input to the system.

**Keywords**—stroke; upper-limb; robot-aided rehabilitation; physiotherapy; shoulder rehabilitation; inverse dynamics control scheme; tracking problem; disturbance rejection;

## I. INTRODUCTION

Stroke is one of the main causes of disability and loss of motor function particularly affecting older people. It affects more than one million people in European Union each year [1]. In the United States more than 0.7 million people become affected by stroke each year [2]. Because of this, using different therapy approaches is necessary to regain motor function and improve functional outcomes. Optimal restoration of arm and hand function is essential to independently perform activities of daily living (ADL). The most common approach in stroke rehabilitation is physiotherapy. This approach is labor-intensive, time-consuming and expensive. Besides, training sessions are often shorter than required for an optimal therapeutic outcome. The therapy varies from one therapist to another and from one hospital to another and is based on theories and therapist's

experience. Furthermore, it lacks repeatability and objective measures of patient performance and progress. Taking all these constraints into consideration, robots can help to improve rehabilitation and become an important tool in stroke rehabilitation. Robot-aided arm therapy is more intensive, of longer duration and more repetitive. By using robots, number and duration of training sessions can be increased, while reducing the number of therapists required per patient. Furthermore, robot-aided therapy provides quantitative measures and supports objective observation and evaluation of the rehabilitation progress. Several studies showed that robot-aided therapy indeed improves motor function more than conventional therapy [3-5]. Lots of researchers around the world have developed some robots for rehabilitation of upper limb. These robots can be classified into end-effector based robots and exoskeleton type robots. Some of end-effector based robots will be stated next. MIT-MANUS [6] developed by Krebs et al was initiated in 1989 and has been in daily operation since 1994. The most prominent feature of this robot is being modular. Unlike most industrial robots, MIT-MANUS has safe and stable operation in close contact with humans. This is achieved by using backdrivable hardware and impedance control. It has low inertia and low friction. The sensorimotor training provided by this robot is by using video games. Mirror Image Motion Enabler (MIME) [7] was initiated in 1998 and was first tested in 2002 by Lum et al. The device is designed for shoulder and elbow neurorehabilitation. Loureiro et al in 2003 introduced GENTLE/s system. GENTLE/s [8] is a system based on haptics and virtual reality visualization techniques. The control system is bead pathway. Hesse et al in 2003 developed a robot-assisted arm trainer, Bi-Manu-Track [9]. The one-DOF device is designed for bilateral passive and active practice of a forearm pronation/supination and wrist flexion/extension. For smooth movement impedance control is implemented. Some exoskeleton type rehabilitation robots will be stated next. Stienen et al in 2007 developed a dynamic force-coordinator trainer for the upper extremities called Dampace [10]. Dampace is a passive exoskeleton that incorporates controlled braking on the three rotational axes of the shoulder and one axis of the elbow. Sanchez et al in 2006 developed T-WREX [11]. T-WREX is actually developed to be a 3D input device to interact with virtual environments.

Carignan and Lizka in 2005 developed an arm exoskeleton named Maryland-Georgetown-Army (MGA) exoskeleton. MGA-exoskeleton [12] incorporates five active degrees of freedom for shoulder and elbow motion. The control schemes used are admittance and impedance control. Frisoli et al in 2007 developed a robotic exoskeleton named L-EXOS [13] system, a force feedback exoskeleton for the right arm. The control scheme used is impedance control. Nef et al from 2003 developed three versions of ARMin robot. Armin I [14] was designed and tested from 2003 to 2006. It had four degrees of freedom, actuating the shoulder in 3D and flexing/extending the elbow with its semi-exoskeleton structure. After ARMin I, ARMin II [15] is developed with a complete exoskeleton structure and two additional degrees of freedom (six altogether). ARMin III [16] is a further improved version of ARMin II in the case of robustness, complexity, user operation and reliability. The control schemes used are PD (Proportional Derivative) control, impedance control and computed torque control. This paper demonstrates mechanical design, simulation and control of a new exoskeleton robot for use in upper-limb rehabilitation after stroke. Initially mechanical design and simulation of the robot in Solidworks is presented and kinematics and dynamics of the robot are derived. Afterwards a control algorithm is proposed and applied for the robot and finally simulation results are presented. The main advantages of this robot compared to similar robots are being light weight, its unique mechanism for third joint, ease of use, more comfort and perfect tracking performance of the controller.

## II. MECHANICAL DESIGN OF THE ROBOT

### A. Mechanical design in Solidworks

After studying the properties of the upper limb of an adult person [17-18] such as mass, moments of inertia and lengths of different segments, an exoskeleton robot is designed for shoulder joint rehabilitation. The Shoulder Rehabilitation System (SRS) has three degrees of freedom for shoulder flexion/extension, abduction/adduction and internal/external rotation. Fig.1. depicts SRS system with a model of a human limb. The system can be wall or wheelchair-mounted. Fig.2. depicts SRS detailed properties. Shoulder flexion/extension is provided by motor 1. Link 1 holds motor 1 from one side and is fastened to the base from the other side. Link 2 which holds motor 2 is L-shaped in order for comfortable accommodation of shoulder joint. Rotation axes of motor 1 and 2 intersect at a point which is shoulder joint placement point. For exact placement of the shoulder joint a wheelchair with adjustable height is recommended. Human shoulder joint does not only have three rotational degrees of freedom but it also possesses translational degrees of freedom. In this robot translational movement of the shoulder joint will not be a problem since that movement is allowed. Motor 2 provides shoulder abduction/adduction. In order to provide shoulder internal/external rotation, the rotation axis of motor 3 cannot be directly aligned with rotation axis of the limb due to anatomical configuration of the upper limb which causes discomfort to the patient. Most robots use gear mechanism

with closed circular configuration. But this will cause discomfort and pressure on the patient's limb. The alternative can be cable mechanism. But using cable mechanism has the drawback of losing connection because of long wiring. By considering these issues a new open circular mechanism is proposed for this robot and power transmission is provided by the gear which is coupled to the shaft of motor 3. Fig.3. depicts details of proposed open circular mechanism for joint 3. Another challenge in exoskeleton robots is that the robot should be adaptable to patient's limb in terms of segment lengths. This issue is not a problem here since the robot segments are of variable lengths.

### B. Kinematics and Dynamics of the robot

Direct kinematics of the robot is derived using Denavit-Hartenberg (DH) convention [19]. Fig.4. depicts link frame assignment and Table I presents the DH parameters of the robot.  $a_i$ ,  $d_i$ ,  $\alpha_i$  and  $\vartheta_i$  are link length, link offset, link twist and joint angle respectively.

The robot is in singular configuration when  $\vartheta_2 = 0$  i.e. when the axis of rotation of the first joint is aligned with the axis of rotation of the third joint. Joint-space control algorithms do not need Jacobian or its inverse so singular points are not a problem. Cartesian space control algorithms need Jacobian or its inverse so singular points should be properly managed. For example in these algorithms one can limit the motion of joint 2 to more than  $10^\circ$  to manage the singular points. Robot dynamics is derived using generalized d'Alembert method [20]. This method gives more efficient equations compared to Lagrange and Newton Euler methods. Besides calculation costs are remarkably reduced in this method and it is ideal for control purposes. Robot dynamics is expressed as follows:

$$\tau = D(q)\ddot{q} + H(q, \dot{q}) + G(q) \quad (1)$$

$D(q)$  is a  $3 \times 3$  inertia matrix,  $H(q, \dot{q})$  is a  $3 \times 1$  matrix of centrifugal and coriolis terms and  $G(q)$  is a  $3 \times 1$  matrix of gravity term.  $\tau$  is a  $3 \times 1$  matrix expressing joint torques.  $q$ ,  $\dot{q}$  and  $\ddot{q}$  are  $3 \times 1$  matrices expressing joint position, velocity and acceleration respectively. These matrices are provided in appendix. Robot dynamics is derived manually and then accredited using Matlab then it is implemented and tested in Matlab and Simulink environment. Table II presents the whole properties of the robot at a glance. For control purposes the state space model of the robot is expressed by the following equations:

$$\dot{X} = f(x) + g(x)\tau \quad (2)$$

$$f(x) = \begin{pmatrix} \dot{q} \\ -D^{-1}(q)(H(q, \dot{q}) + G(q)) \end{pmatrix} \quad (3)$$

$$g(x) = \begin{pmatrix} O_{n \times n} \\ D^{-1}(q) \end{pmatrix} \quad (4)$$

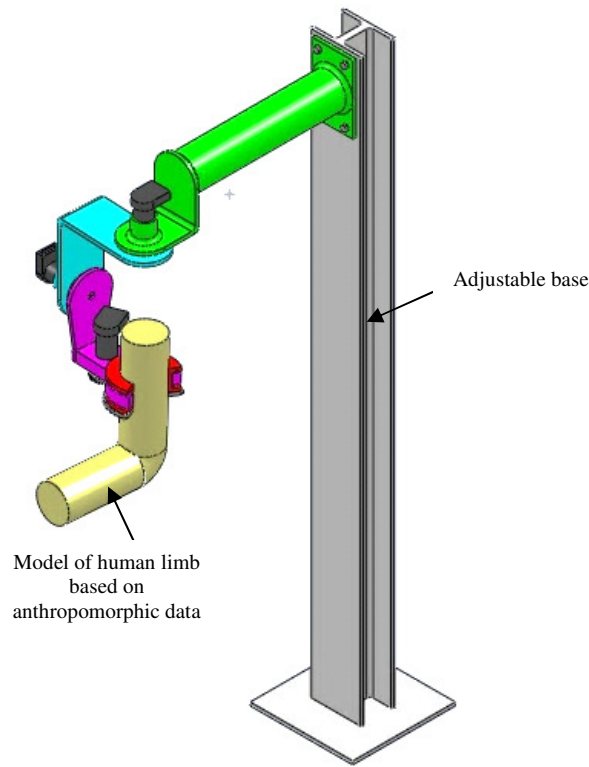


Figure 1. Shoulder rehabilitation system with a model of a human limb

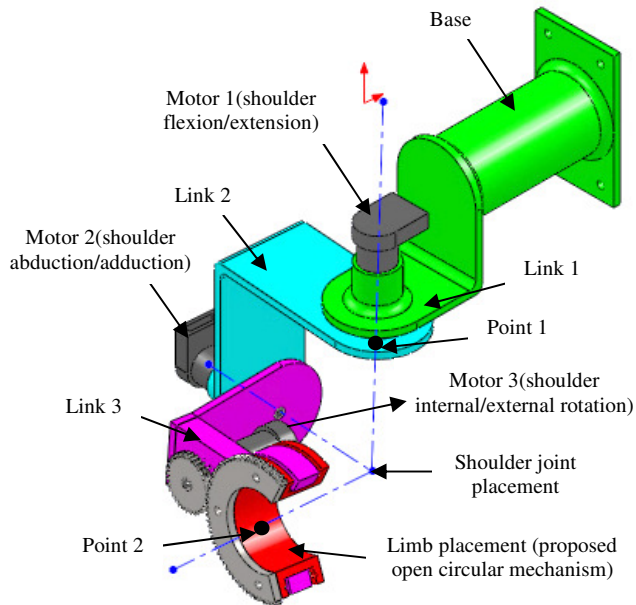


Figure 2. Detailed properties of shoulder rehabilitation system

### III. CONTROLLER DESIGN

Control algorithms implemented on rehabilitation robots are designed with two major aims: (1) passive rehabilitation in which the patient remains passive and the robot moves the

patient's hand through a predefined trajectory and (2) active rehabilitation in which the patient initiates the movement and is partially assisted or resisted by the robotic device. In this paper the designed controller is intended to be used in passive rehabilitation. The proposed controller is tracking inverse dynamics controller with integral action [19].

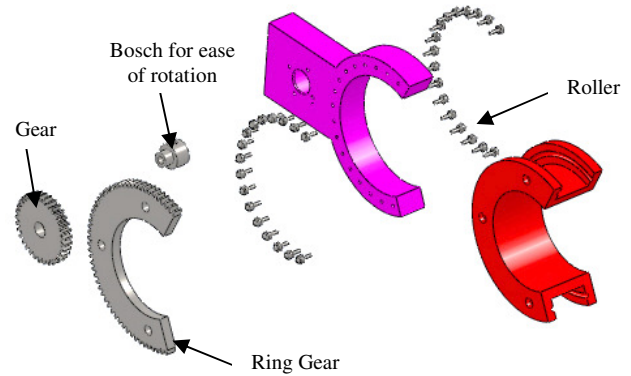


Figure 3. Details of proposed circular mechanism

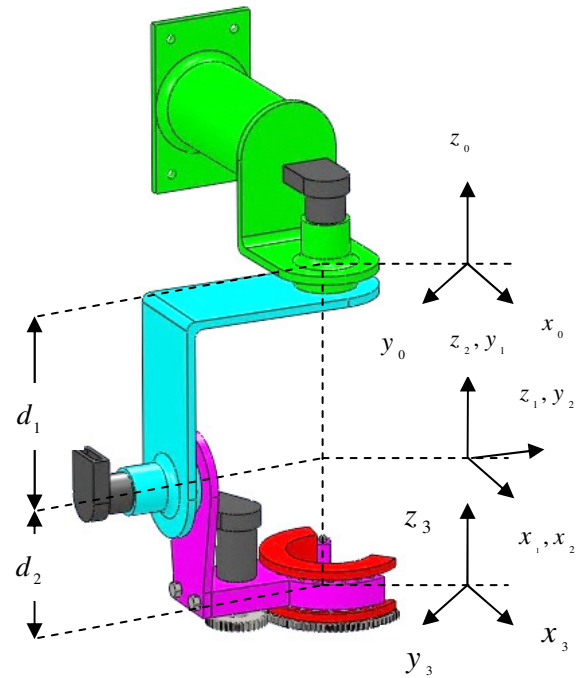


Figure 4. Link frame assignments in DH convention

TABLE I. DH PARAMETERS OF THE ROBOT

Joint i	$a_i$	$d_i$	$\alpha_i$	$\vartheta_i$
1	0	$d_1$	$-\pi/2$	$\vartheta_1$
2	0	0	$\pi/2$	$\vartheta_2$
3	0	$d_2$	0	$\vartheta_3$

TABLE II. MECHANICAL PROPERTIES OF THE ROBOT AT A GLANCE

Body segment	Segment length (m)	Segment weight (kg)	Moments of inertial (kg.m <sup>2</sup> ) with respect to center of mass and expressed in the center of mass coordinate frame			Center of mass (m) expressed in the base coordinate frame		
			I <sub>xx</sub>	I <sub>yy</sub>	I <sub>zz</sub>	x	y	z
Shoulder joint(from point 1 in Fig.2. to shoulder joint placement)	0.21	3.8	0.0934	0.0511	0.0559	-0.193	0.0096	-0.174
Arm (from shoulder joint placement to point 2 in Fig.2.)	0.12	3.6	0.0728	0.0148	0.0164	-0.111	0.00471	-0.337
Limb and limb holder	0.035	4.5	0.0175	0.0621	0.0719	-0.0067	-0.044	-0.447

In order to properly track the desired trajectory the proposed controller should have the disturbance rejection property. The disturbance here is considered to be constant and bounded and all other system uncertainties are modeled as a constant bounded disturbance. Suppose  $d$  is the disturbance torque. System dynamic model can be stated as follows:

$$D(q)\ddot{q} + H(q, \dot{q}) + G(q) = \tau + d \quad (5)$$

The proposed control action is as follows:

$$\tau = D(q)\nu + H(q, \dot{q}) + G(q) \quad (6)$$

$$\nu = \ddot{q}_d + K_D(\dot{q}_d - \dot{q}) + K_P(q_d - q) + K_I \int_0^t (q_d - q) ds \quad (7)$$

Let us define the error vector and its first and second derivatives as:

$$e = q - q_d \rightarrow \dot{e} = \dot{q} - \dot{q}_d \rightarrow \ddot{e} = \ddot{q} - \ddot{q}_d \quad (8)$$

By substituting (6) and (7) in (5) and by using (8) one can write:

$$\ddot{e} + K_D\dot{e} + K_P e + K_I \int e dt = \delta \quad (9)$$

$$\delta = D^{-1}(q)d \quad (10)$$

In which  $K_P = k_p I_{3 \times 3}$ ,  $K_D = k_d I_{3 \times 3}$  and  $K_I = k_i I_{3 \times 3}$  are three positive definite matrices. Fig.5. depicts the block diagram of the closed loop system. Integral term in this controller has the property of rejecting disturbance. To prove this, by rewriting (9) for each joint one can obtain:

$$e(s) = \frac{\delta_i}{s^3 + k_d s^2 + k_p s + k_i}, i = 1, 2, 3 \quad (11)$$

Using final value theorem the steady state error can be defined as follows:

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s e(s) = \lim_{s \rightarrow 0} \frac{s}{s^3 + k_d s^2 + k_p s + k_i} \delta_i = 0 \quad (12)$$

So the proposed controller can efficiently reject disturbance and other bounded system uncertainties.

#### IV. SIMULATION AND RESULTS

For simulation the desired trajectory which does not include singular point of the system is defined as:

$$q_d = \begin{bmatrix} 0.5 \sin(t) \\ 0.5 \cos(t) + 1 \\ 0.5 \cos(t) \end{bmatrix} \quad (13)$$

And  $k_p = 42$ ,  $k_d = 21$  and  $k_i = 21$ . The simulation duration is 20(s) and the input disturbance is considered to be constant and bounded. Fig.6. depicts simulation results with this controller in tracking desired trajectory for each joint. Fig.7. depicts control inputs and tracking errors are depicted in Fig.8. It is clear that the tracking error is small and the controller is efficient in tracking desired trajectory.

#### V. CONCLUSION

In this paper mechanical design, simulation and control of a new exoskeleton robot for use in upper-limb rehabilitation after stroke was presented. The mechanical design was done and tested in Solidworks and kinematics and dynamics of the robot were derived using DH convention and generalized d'Alembert method respectively. A new circular open mechanism was proposed for joint 3. A control algorithm based on inverse dynamics control with integral action was proposed and implemented on the robot. Simulation results with this controller showed effectiveness of this controller in tracking desired trajectory and rejecting constant bounded disturbance.

#### VI. FUTURE WORK

After obtaining successful results in the CAD environment and effectiveness of the controller in tracking desired trajectories, the robot will be made and experimental results will be published in the future.

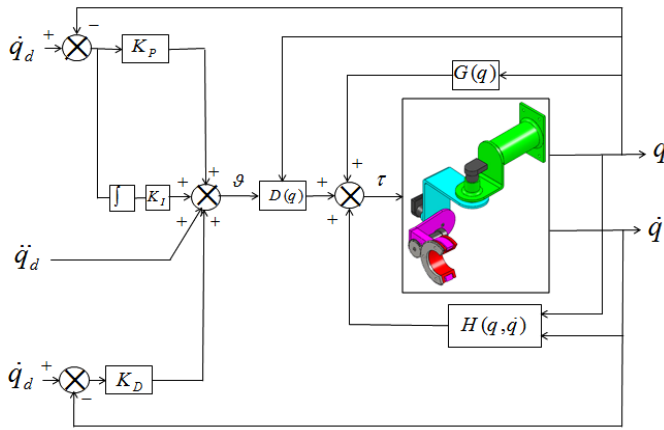


Figure 5. Block diagram of the closed loop system with inverse dynamics controller

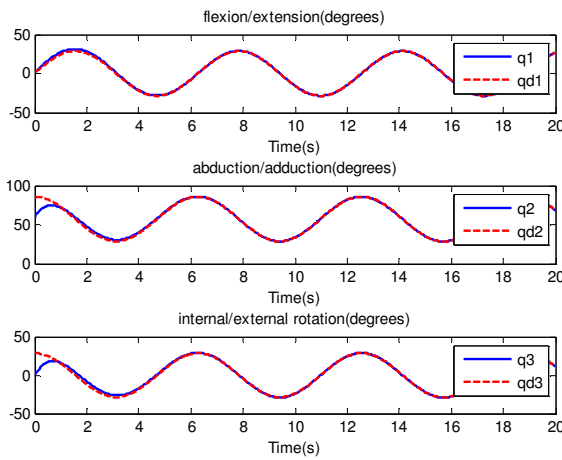


Figure 6. Simulation results with inverse dynamics controller in tracking desired trajectory for each joint

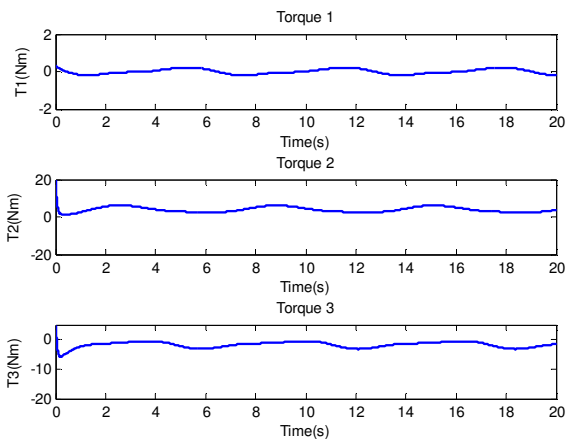


Figure 7. Control inputs based on inverse dynamics controller for each joint

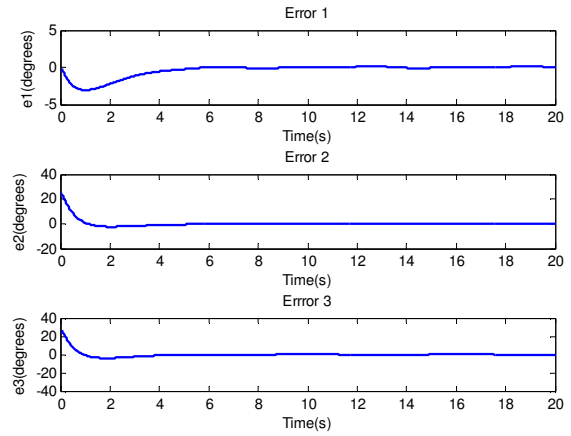


Figure 8. Tracking error based on inverse dynamics controller for each joint

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## A. APPENDIX

In (1):

$$D(q) = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{pmatrix}, H(q, \dot{q}) = \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}, G(q) = \begin{pmatrix} G_1 \\ G_2 \\ G_3 \end{pmatrix}$$

Let us define  $m_i, i=1,2,3$  link masses and inertia tensor matrix as:

$$I_i = \begin{pmatrix} I_{ixx} & 0 & 0 \\ 0 & I_{iyy} & 0 \\ 0 & 0 & I_{izz} \end{pmatrix}, i = 1, 2, 3$$

Using Table I transformation matrices will be derived as follows:

$$T_1^0 = \begin{pmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, T_2^1 = \begin{pmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, T_3^2 = \begin{pmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where  $c_i$  and  $s_i$  are  $\cos(\vartheta_i)$  and  $\sin(\vartheta_i)$  respectively. Let

$P_i$  be the last column of  $T_i^0$

and  $r_i = (r_{ix} \ r_{iy} \ r_{iz})^T, i = 1, 2, 3$  is link center of masses vector. By defining  $\bar{c}_i = r_i - P_{i-1}, i = 1, 2, 3$  one can obtain:

$$\begin{aligned} D_{11} &= I_{1yy} + m_1(r_{1x}^2 + r_{1y}^2) + m_2(r_{2x}^2 + r_{2y}^2) + m_3(r_{3x}^2 + r_{3y}^2) + \\ & (I_{2zz} + I_{3zz})c_2^2 + (I_{2xx} + I_{3xx}c_3^2 + I_{3yy}s_3^2)s_2^2 \\ D_{12} &= (I_{3yy} - I_{3xx})s_2s_3c_3 + m_2(r_{2x}c_{2z}s_1 - r_{2y}c_{2z}c_1) + \\ & m_3(r_{3x}c_{3z}s_1 - r_{3y}c_{3z}c_1) \end{aligned}$$

$$\begin{aligned} D_{13} &= I_{3zz}c_2 + m_3((r_{3x}c_{3x} + r_{3y}c_{3y})c_2 - (r_{3x}c_{3z}c_1 + r_{3y}c_{3z}s_1)s_2) \\ D_{22} &= I_{2yy} + I_{3xx} + (I_{3yy} - I_{3xx})c_3^2 + m_2(c_{2y}^2 + c_{2z}^2 + c_1^2(c_{2x}^2 - c_{2y}^2) \\ & + 2c_{2x}c_{2y}c_1s_1) + m_3(c_{3y}^2 + c_{3z}^2 + 2c_{3x}c_{3y}c_1s_1 + c_1^2(c_{3x}^2 - c_{3y}^2)) \\ D_{23} &= m_3((c_{3x}^2s_1c_1 - 2c_{3x}c_{3y}c_1^2 + c_{3x}c_{3y} - c_{3y}^2s_1c_1)s_2 + (c_{3x}c_{3z}s_1 - c_{3y}c_{3z}c_1)c_2) \\ D_{33} &= I_{3zz} + m_3(c_{3x}^2(1 - c_1^2s_2^2) - 2c_{3x}c_{3y}c_1s_1s_2^2 - 2c_{3x}c_{3z}c_1c_2s_2 + \\ & c_{3y}^2c_1^2s_2^2 + c_{3y}^2c_2^2 - 2c_{3y}c_{3z}s_1s_2c_2 + c_{3z}^2s_2^2) \end{aligned}$$

$$\begin{aligned} H_1 &= \dot{\vartheta}_1^2(m_1(c_{1x}r_{1y} - c_{1y}r_{1x}) + m_2(r_{2y}c_{2x} - r_{2x}c_{2y}) + \\ & m_3(c_{3x}r_{3y} - c_{3y}r_{3x})) + \dot{\vartheta}_1\dot{\vartheta}_2(2(I_{2xx} - I_{2zz} + I_{3yy} - I_{3zz})c_2s_2 + \\ & 2m_2(r_{2x}c_{2z}c_1 + r_{2y}c_{2z}s_1) + 2m_3(r_{3x}c_{3z}c_1 + r_{3y}c_{3z}s_1)) + \\ & \dot{\vartheta}_1\dot{\vartheta}_3(2(I_{3yy} - I_{3xx})c_3s_3s_2^2 + m_3(c_{3x}r_{3y} - r_{3x}c_{3y} + \\ & r_{3y}c_{3x}c_2 - c_{3y}r_{3x}c_2 - c_{3z}r_{3y}c_1s_2 + c_{3z}r_{3x}s_1s_2)) + \\ & \dot{\vartheta}_2^2(m_2(r_{2y}c_{2x}c_1^2 - r_{2x}c_{2y}s_1^2 + c_1s_1(r_{2y}c_{2y} - r_{2x}c_{2x})) + \\ & m_3(r_{3y}c_{3x}c_1^2 - r_{3x}c_{3y}s_1^2 + c_1s_1(r_{3y}c_{3y} - r_{3x}c_{3x})) + \\ & (I_{3yy} - I_{3xx})c_2c_3s_3) + \dot{\vartheta}_2\dot{\vartheta}_3((I_{3xx} - I_{3yy} - I_{3zz})s_2 + \\ & 2(I_{3yy} - I_{3xx})s_2c_3^2 + m_3((c_{3y}r_{3x} + c_{3x}r_{3y})c_1s_1s_2 - \\ & c_{3x}r_{3x}s_1^2s_2 - c_{3y}r_{3y}c_1^2s_2)) + \dot{\vartheta}_3^2(m_3c_2(r_{3y}c_{3x} - r_{3x}c_{3y})) \\ H_2 &= \dot{\vartheta}_1^2((I_{2zz} - I_{2xx} + I_{3yy} + I_{3zz} - I_{3xx}c_3^2 + I_{3yy}c_3^2)c_2s_2 - \\ & m_2c_{2z}(c_{2y}s_1 + c_{2x}c_1) - m_3c_{3z}(c_{3x}c_1 + c_{3y}s_1)) + \\ & \dot{\vartheta}_1\dot{\vartheta}_3((I_{3xx} - I_{3yy} + I_{3zz} - 2I_{3xx}c_3^2 + 2I_{3yy}c_3^2)s_2 + \\ & 2m_3(c_{3z}^2s_2 - c_{3x}c_{3z}c_1c_2 - c_{3y}c_{3z}c_1s_1)) + \\ & \dot{\vartheta}_3^2(m_3((c_{3z}^2 - c_{3y}^2 - c_{3x}^2c_1^2 + c_{3y}^2c_1^2)c_2s_2 + \\ & c_{3y}c_{3z}s_1 + c_{3x}c_{3z}c_1 - 2c_{3x}c_{3z}c_1c_2^2 - 2c_{3y}c_{3z}c_1^2s_1 - \\ & c_{3x}c_{3y}c_1c_2s_1s_2)) + \dot{\vartheta}_2\dot{\vartheta}_3(2(I_{3xx} - I_{3yy})c_3s_3 + \\ & 2m_3(c_{3x}c_{3y}c_2 + c_{3y}c_{3z}c_1s_2 - c_{3x}c_{3z}s_1s_2 - \\ & 2c_{3x}c_{3y}c_1^2c_2 + c_{3x}^2c_1c_2s_1 - c_{3y}^2c_1c_2s_1)) \\ H_3 &= \dot{\vartheta}_1^2((I_{3xx} - I_{3yy})c_3s_2^2s_3 + m_3(c_{3y}c_{3z}c_1s_2 - c_{3x}c_{3z}s_1s_2)) + \\ & \dot{\vartheta}_1\dot{\vartheta}_2((I_{3xx} - I_{3yy} - I_{3zz} - 2I_{3xx}s_3^2 + 2I_{3yy}s_3^2)s_2 + \\ & 2m_3c_{3z}(c_{3y}c_2s_1 + c_{3x}c_1c_2 - c_{3z}s_2)) + \dot{\vartheta}_2^2((I_{3yy} - I_{3xx})c_3s_3 + \\ & m_3((c_{3y}^2 - c_{3x}^2)c_1s_1c_2 + c_{3x}c_{3y}c_1^2c_2 + \\ & c_{3x}c_{3z}s_1s_2 - c_{3y}c_{3z}c_1s_2 - c_{3x}c_{3y}c_2)) \end{aligned}$$

$$G_1 = 0$$

$$G_2 = -g_0(m_2(c_{2x}c_1 + c_{2y}s_1) + m_3(c_{3x}c_1 + c_{3y}s_1))$$

$$G_3 = g_0m_3s_2(-c_{3x}s_1 + c_{3y}c_1)$$